

Variable-Structure Control of Spacecraft Large-Angle Maneuvers

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The spacecraft large-angle maneuver problem is treated using the principles of variable-structure control theory. A control law that constrains the state to follow a specified path (the so-called sliding mode) in the state-space is designed on the basis of a simplified model of the spacecraft dynamics. The sliding mode is obtained by solving an optimal control problem posed in a reduced space, the solution being the angular velocities of the spacecraft as functions of the attitude variables (Euler parameters or quaternions in the present context). It is shown that the motion along the sliding mode is insensitive to parameter variations and unmodeled effects.

Introduction

THRUSTERS and reaction wheels are commonly used spacecraft attitude control devices. Thrusters are used for rapid large-angle maneuvers and a coarse attitude hold mode. Reaction wheels are used for slow large-angle maneuvers and to maintain a precise attitude (on-orbit normal mode).

Feedback control laws for large-angle maneuvers have been obtained for reaction wheel and continuous external torque systems using optimal control theory^{1,2} and Liapunov's second method.^{3,4} Recently, Wie and Barba⁵ analyzed various feedback control schemes for reaction jet systems with pulse-width and pulse-frequency (PWPF) modulation. The PWPF modulator can provide an average output torque to equal a demanded, continuous profile, if operated in the linear range. A three-axis stabilized, minimum-time, attitude controller design using Euler parameters has been given by Burdick et al.⁶ Salehi and Ryan⁷ presented nonlinear optimal control schemes for regulating spacecraft angular momentum using reaction wheels and on-off thrusters. The judicious use of both these systems can provide improved performance and fuel economy, as well as redundancy and capability to perform auxiliary functions (e.g., momentum dumping).

In this paper, the principles of variable-structure control theory^{8,9} are applied to the large-angle maneuver problem. Control systems of variable structure employ different control laws during the control process to enhance the system performance (e.g., transient response and robustness) in comparison to fixed-structure control systems. Variable-structure controls can be implemented by changing from negative to positive feedback during finite intervals of the control process (e.g., a bang-bang control law) or by switching (off or on) primary and secondary feedbacks, for example. (For applications of variable-structure control theory to aerospace systems, see Refs. 10 and 11.) Most of the attention in this area has been focused on designing control laws that switch at theoretically infinite frequencies. The trajectory in the state-space then moves along the intersection of a set of switching surfaces (sliding manifold); this motion has come to be known as "ideal sliding." In practice, because of computational delays, noise, inertia of the switching components (relays), and unmodeled dynamics, switching takes place at a finite (high)

frequency, often giving rise to chattering controls. In the sliding regime, the system is robust with respect to parameter variations and external disturbances.

The variable-structure control design process is done in three stages. First, switching surfaces with desired properties are selected. Next, control laws are designed that will guarantee the existence of sliding modes on the switching surfaces. Finally, it is ensured that the state trajectory can be forced toward the sliding manifold from any initial state. The above procedure is applied to the spacecraft large-angle maneuver problem. For spacecraft applications, control chatter is undesirable. Although a deadband is generally included in the control system, depending on the switching surfaces and thrust magnitude, chattering may occur along the edges of the deadband. To avoid chattering and still retain the advantages of variable-structure control, we propose a control scheme that requires the thrusters to be operated first in a continuous manner until the state trajectory reaches the sliding manifold and then in pulse mode until the fine pointing stages of the maneuver, where reaction wheels are used as the primary control elements.

Spacecraft Dynamics and Kinematics

The equations of rotational motion of a rigid spacecraft are

$$\begin{aligned} I_1 \dot{\omega}_1 &= (I_2 - I_3) \omega_2 \omega_3 + L_1 \\ I_2 \dot{\omega}_2 &= (I_3 - I_1) \omega_3 \omega_1 + L_2 \\ I_3 \dot{\omega}_3 &= (I_1 - I_2) \omega_1 \omega_2 + L_3 \end{aligned} \quad (1)$$

where ω_1 , ω_2 , and ω_3 are the spacecraft angular velocities, I_1 , I_2 , and I_3 the spacecraft principal moments of inertia, and L_1 , L_2 , and L_3 the external torques.

The evolution of the spacecraft orientation in terms of Euler parameters ($\beta_0, \beta_1, \beta_2, \beta_3$) is given by

$$\dot{\beta} = \frac{1}{2} [G(\omega)] \beta \quad (2)$$

where

$$[G(\omega)] = \begin{bmatrix} 0 & -\omega_1 & -\omega_2 & -\omega_3 \\ \omega_1 & 0 & \omega_3 & -\omega_2 \\ \omega_2 & -\omega_3 & 0 & \omega_1 \\ \omega_3 & \omega_2 & -\omega_1 & 0 \end{bmatrix}$$

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It is well known that the Euler parameters are constrained by the relation

$$\beta^T \beta = 1 \quad (3)$$

Selection of Switching Surfaces

Under ideal sliding conditions, the trajectory in the state-space moves on the sliding manifold. Such constrained motion can be described by a smaller number of variables than necessary to describe the actual system dynamics. The reduction in the number of variables is equal to the number of constraints or the number of switching surfaces. For example, if the spacecraft angular velocities are constrained to be explicit functions of the spacecraft attitude, the four variables of the vector β are sufficient to describe the motion. With this in mind, the following reduced-order, optimal control problem is posed:

Find a control law of the form

$$\omega = \omega(\beta) \quad (4)$$

to minimize the performance index

$$J = \frac{1}{2} \int_{t_s}^{\infty} [\beta^T Q \beta + \omega^T R \omega] dt \quad (5)$$

subject to the bilinear system constraint given by Eq. (2). Note that the trajectory may arrive at different switching surfaces at different times, but t_s is the time of arrival at the sliding manifold. Q and R are symmetric weighting matrices. The Hamiltonian is written as

$$H = \frac{1}{2} (\beta^T Q \beta + \omega^T R \omega) + \lambda^T \dot{\beta} \quad (6)$$

where λ is the vector of costates. The necessary conditions for optimality are

$$\dot{\lambda} = -Q\beta - \frac{1}{2} G^T \lambda \quad (7)$$

and

$$\omega = -\frac{1}{2} R^{-1} K^T(\beta) \lambda \quad (8)$$

where

$$K(\beta) = \begin{bmatrix} -\beta_1 & -\beta_2 & -\beta_3 \\ \beta_0 & -\beta_3 & \beta_2 \\ \beta_3 & \beta_0 & -\beta_1 \\ -\beta_2 & \beta_1 & \beta_0 \end{bmatrix}$$

It is of interest to note that Eq. (2) can also be written as

$$\dot{\beta} = \frac{1}{2} K(\beta) \omega \quad (9)$$

When ω is eliminated from Eq. (2) by using Eq. (8), the following state equation is obtained:

$$\dot{\beta} = -\frac{1}{4} K(\beta) R^{-1} K^T(\beta) \lambda \quad (10)$$

The solution for the unknown costate vector λ must satisfy Eqs. (10) and (7) simultaneously. At the final time, the angular velocity vector should approach zero for slewing maneuvers. It is assumed, without any loss of generality, that the final orientation is given by $\beta = [1, 0, 0, 0]^T$. Inspection of Eq. (8) suggests the following choice for the costate vector:

$$\lambda = \lambda(e) \quad (11)$$

where

$$e = [(\beta_0 - 1), \beta_1, \beta_2, \beta_3]^T$$

Using Eqs. (7), (10), and (11), a functional equation for λ is obtained,

$$\frac{1}{4} \left[\frac{\partial \lambda}{\partial e} \right] K R^{-1} K^T \lambda = Q\beta + \frac{1}{2} G^T \lambda \quad (12)$$

An approximate solution to the above equation can be found by assuming λ to be a polynomial function in the e_i as shown in Ref. 1. Under the simplifying assumption of

$$Q = \text{diag}[0, q, q, q]$$

where q is a positive scalar and R the identity matrix, the following analytical result is obtained for λ :

$$\lambda = 2\sqrt{q} e \quad (13)$$

To show that Eq. (13) satisfies Eq. (12), the left- and right-hand sides of Eq. (12) are evaluated separately,

$$\text{LHS} = q K K^T e \quad (14)$$

$$\text{RHS} = Q\beta + q K K^T e + \sqrt{q} \begin{bmatrix} 0 \\ \omega \end{bmatrix} \quad (15)$$

Hence, it needs to be shown that

$$Q\beta = -\sqrt{q} \begin{bmatrix} 0 \\ \omega \end{bmatrix} \quad (16)$$

Substituting Eq. (13) into Eq. (8), ω is obtained as

$$\omega = -\sqrt{q} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} = -\sqrt{q} \tilde{\beta} \quad (17)$$

where $\tilde{\beta}$ is the reduced Euler parameter vector. From Eq. (17) and the structure of Q , it can be easily verified that Eq. (16) is satisfied. Hence, the optimal switching surfaces are given by

$$S_i = \omega_i + k\beta_i = 0, \quad i = 1, 2, 3 \quad (18)$$

where $k = +\sqrt{q}$.

For this special case, it can also be shown that the optimal cost of regulation [the value of the integral in Eq. (5)] is given by

$$J^* = 2k[1 - \beta_0(t_s)] \quad (19)$$

Even though β and $-\beta$ represent the same attitude, notice that the optimal regulation cost given by Eq. (19) for each case is different; specifically, if $\beta_0(t_s)$ is negative, the regulation cost is more. In order to remove this ambiguity, the attitude error vector can be redefined as

$$e = [\beta_0 - \text{sgn}[\beta_0(t_s)], \beta_1, \beta_2, \beta_3]^T \quad (20)$$

leading to

$$S_i = \omega_i + k\beta_i \text{sgn}[\beta_0(t_s)] = 0, \quad i = 1, 2, 3 \quad (21)$$

and

$$J^* = 2k[1 - |\beta_0(t_s)|] \quad (22)$$

The final orientation will then correspond to a β_0 of either $+1$ or -1 depending on $\beta_0(t_s)$.

The sliding motion is described by the following differential equations obtained by eliminating ω from Eq. (2) by using

Eq. (21):

$$\dot{\beta}_0 = \frac{1}{2}k[1 - \beta_0^2] \operatorname{sgn}[\beta_0(t_s)] \quad (23a)$$

$$\dot{\beta}_i = -\frac{1}{2}k\beta_i\beta_0 \operatorname{sgn}[\beta_0(t_s)], \quad i = 1, 2, 3 \quad (23b)$$

From Eq. (23a), the solution for the Euler principal rotation angle, $\phi = 2\cos^{-1}(\beta_0)$ is obtained as

$$\tan \frac{\phi}{4} = \tan \left[\frac{\phi(t_s)}{4} \right] \exp \left[\pm \frac{1}{2}k(t - t_s) \right] \quad (24)$$

It is interesting to note that Eqs. (23) imply that the Euler principal axis of rotation remains fixed. The \pm sign in Eq. (24) is included to account for the desired final orientation of $\phi = 0$ or 360 deg. Equations (23) do not depend on any of the spacecraft parameters (namely, moments of inertia). Although a large value of k is desirable for a fast transient response, as shown in the next section, the control magnitude increases with k . Due to saturation constraints on the control magnitude, it may not be possible to maintain the sliding motion if k is too large. From Eq. (23a), it is also seen that the derivative $\dot{\beta}_0$ is either positive or negative depending on the sign of $\beta_0(t_s)$. Hence, without any loss of generality the term $\operatorname{sgn}[\beta_0(t_s)]$ in Eqs. (21) and (23) can be replaced by $\operatorname{sgn}(\beta_0)$.

Control Law Design

Controls that induce ideal sliding are obtained by the "equivalent control method."⁸ The equivalent control is an idealization of the chattering control that keeps the state trajectory in the vicinity of the sliding manifold. It is obtained by assuming that the switching frequency is infinite and the state slides on the manifold. A system of n equations, linear in the m controls, is

$$\dot{x} = f(x) + BL \quad (25)$$

where B is a constant $n \times m$ matrix. Let

$$S = [S_1, S_2, \dots, S_m]^T = 0$$

denote the sliding manifold. During ideal sliding on $S = 0$, \dot{S} can be set to zero,

$$\dot{S} = \left[\frac{\partial S}{\partial x} \right] \dot{x} = Gf(x) + GBL_{eq} = 0 \quad (26)$$

where $G = [\partial S / \partial x]$, an $m \times n$ matrix. Thus, the equivalent control is

$$L_{eq} = -[GB]^{-1}Gf \quad (27)$$

If the state trajectory can reach the sliding manifold, the equivalent control can keep it there. The equivalent control obtained above can be thought of as the mean value of the discontinuous control that induces the sliding motion. The existence and uniqueness of the equivalent control depends on the matrix $[GB]$. Under the effects of external disturbances, unmodeled dynamics, and parameter variations, the equivalent control derived above may not be able to maintain the sliding motion. Hence, augmentation of L_{eq} is necessary to account for the above-mentioned nonideal effects such that the state remains near the sliding manifold. Assuming that two non-negative coefficients α_{1i} and α_{2i} can be found for each spacecraft axis such that

$$\begin{aligned} |L_{iU}| &< \alpha_{1i}|S_i| \\ |L_{iD}| &< \alpha_{2i} \end{aligned} \quad (28)$$

where L_{iU} and L_{iD} are estimates of the worst-case torques on the design model due to unmodeled effects and external disturbances, respectively. The sliding mode controls can be selected as

$$L = -[GB]^{-1}Gf - [\alpha_1]S - [\alpha_2]\operatorname{sgn}(S) \quad (29)$$

where $[\alpha_1] = \operatorname{diag}[\alpha_{1i}]$ and $[\alpha_2] = \operatorname{diag}[\alpha_{2i}]$.

The sliding mode equations are then given by

$$\dot{S} = -[GB]\{[\alpha_1]S + [\alpha_2]\operatorname{sgn}(S)\} \quad (30)$$

If $[GB]$ is positive definite, then the sliding mode coordinates are asymptotically stable. A deadband should be included around the switching surfaces to provide a coarse attitude hold mode. To avoid chattering, the coefficients α_{2i} should be as small as possible. Near the switching surfaces, the coefficients α_{1i} have to be large, as the S_i are very small.

For the problem at hand, the control design is based on a simplified model of the spacecraft obtained by deliberately neglecting the dynamic nonlinearities in the Euler equations,

$$I_i \dot{\omega}_i = L_i, \quad i = 1, 2, 3 \quad (31)$$

With

$$x = \begin{bmatrix} \beta \\ \omega \end{bmatrix}, \quad f = \begin{bmatrix} \dot{\beta} \\ 0_{3 \times 1} \end{bmatrix}, \quad B = \begin{bmatrix} 0_{4 \times 3} \\ \operatorname{diag}[I_i]^{-1} \end{bmatrix}$$

$$s = \omega + k\tilde{\beta} \operatorname{sgn}(\beta_0)$$

$$G = [0_{3 \times 1} \quad k \operatorname{sgn}(\beta_0) I_{3 \times 3} \quad I_{3 \times 3}]$$

the sliding mode controls can be found to be

$$L_i = -I_i k \omega_i |\beta_0| / 2 - \alpha_{1i} S_i - \alpha_{2i} \operatorname{sgn}(S_i) \quad i = 1, 2, 3 \quad (32)$$

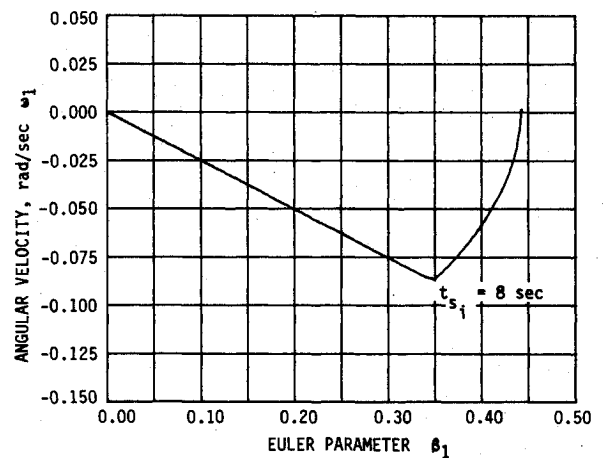
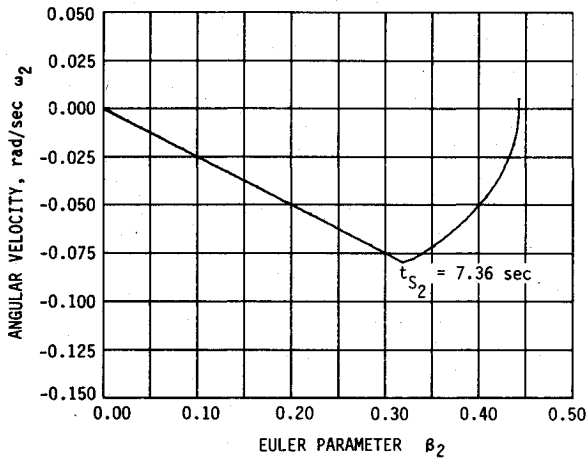
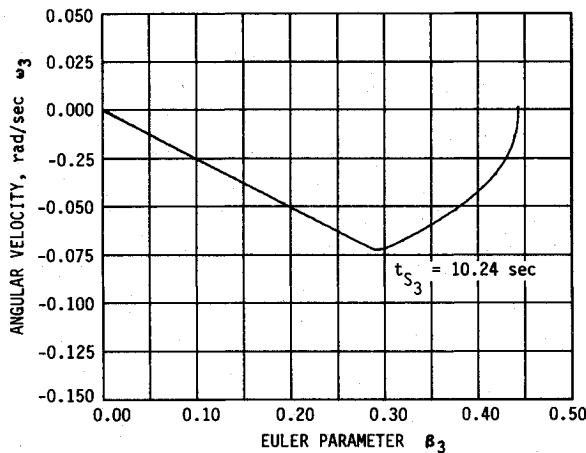


Fig. 1 State trajectory in the β_1 - ω_1 plane.

Table 1 Boundary conditions

State	Initial conditions	Final conditions
β_0	0.6428	1
β_1	0.4423	0
β_2	0.4423	0
β_3	0.4423	0
ω_1	0.001 rad/s	0
ω_2	0.005 rad/s	0
ω_3	0.001 rad/s	0

Fig. 2 State trajectory in the β_2 - ω_2 plane.Fig. 3 State trajectory in the β_3 - ω_3 plane.

For the sake of dimensional consistency, it should be noted that the units of k , α_{1i} , and α_{2i} should be rad/s, N·m/s, and N·m, respectively.

Next, the controls that can force the state trajectories toward the sliding manifold are obtained. For this purpose, a suitable Liapunov function for the closed loop system is selected as

$$V = \frac{1}{2} S^T S \quad (33)$$

The time derivative of V is

$$\dot{V} = S^T \dot{S} = S^T \left[\dot{\omega} + k \dot{\beta} \text{sgn}(\beta_0) \right] \quad (34)$$

Note that Eqs. (23) are not valid when the state trajectories are far from the switching surfaces; then, Eq. (2) should be used instead. Substituting for $\dot{\omega}$ and $\dot{\beta}$ from Eq. (31) and (2), respectively, in Eq. (34) and including the unmodeled and disturbance torques, one obtains

$$\dot{V} = \sum_{i=1}^3 S_i \left[\frac{L_i + L_{U_i} + L_{D_i}}{I_i} + \frac{1}{2} k \omega_i |\beta_0| \right] \quad (35)$$

It can be seen by direct substitution that the controls given by Eq. (32), along with the condition of Eq. (28), render \dot{V} negative definite. Hence, the sliding mode controls can also force the state trajectories toward the sliding manifold asymptotically.

For fast transition from the initial conditions to the sliding manifold, the thrusters may have to be operated first in a

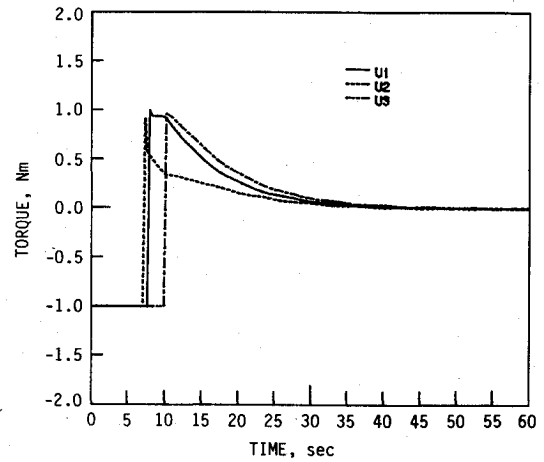


Fig. 4 Control torques.

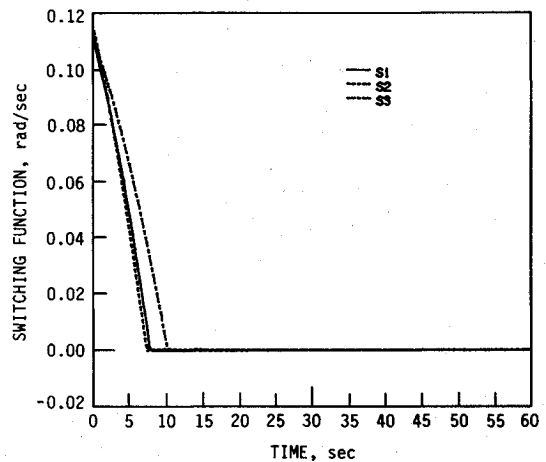


Fig. 5 Switching functions.

continuous manner (full thrust) until sliding begins and then in pulse mode⁵ to provide sliding mode controls. If the required sliding mode torques are small enough (< 0.05 N·m), reaction wheels can be used instead of thrusters. At the end of the maneuver, a fine pointing control law using reaction wheels¹² can be initiated for the on-orbit normal mode.

Numerical Example

An example of a three-axis maneuver is presented. The initial and final conditions are shown in Table 1. The nominal values of I_1 , I_2 , and I_3 used for control design are 86, 85, and 113 kg/m², respectively.

The values of I_1 , I_2 , and I_3 used for the simulation are off from their nominal values by 10, -5, and 15%, respectively. The maximum torque about each axis is limited to 1 N·m. The constant k specifying the switching surfaces is selected to be 0.25. The disturbance torques are assumed to be

$$L_{1D} = -0.005 \sin(\omega t) \text{ N} \cdot \text{m}$$

$$L_{2D} = 0.005 \sin(\omega t) \text{ N} \cdot \text{m}$$

$$L_{3D} = -0.005 \sin(\omega t) \text{ N} \cdot \text{m}$$

where ω is the disturbance frequency, assumed to be 1 rad/s. The coefficients α_{11} , α_{12} , and α_{13} are chosen as $10I_1$, $10I_2$, and $2I_3$, respectively, to compensate for the neglected nonlinearities. From the nominal moment of inertia data, it is apparent that this modeling error has the least effect on the spacecraft three-axis and hence α_{13} is chosen to be less than α_{11} and α_{12} . The coefficients α_{2i} are chosen as 0.005, based on

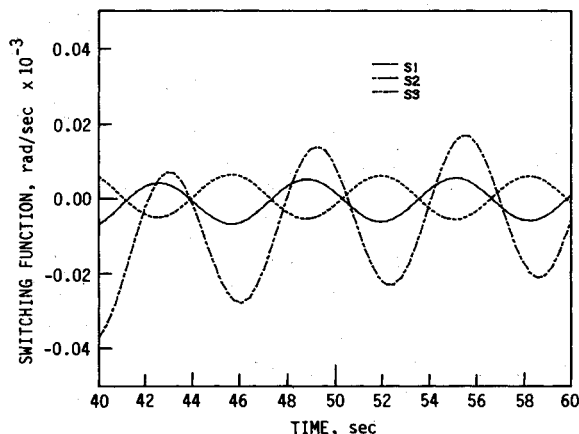


Fig. 6 Switching functions during the terminal maneuver.

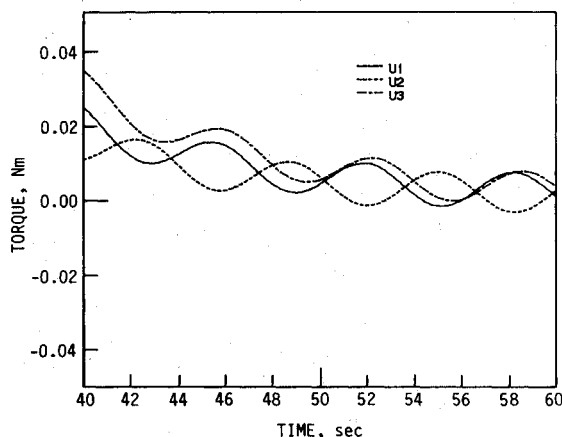


Fig. 7 Control torques during the terminal maneuver.

the disturbance torque magnitudes. A deadband of ± 0.1 deg is included for each switching surface. In terms of the switching function, this corresponds to $S_i = \pm 0.000218$ rad/s.

Figures 1-3 show the state trajectory projected onto the (β_i, ω_i) subspace for each spacecraft axis. Also indicated on the figures are the arrival times (t_{s_i}) at the respective switching surfaces. Figure 4 shows the control torques commanded during the maneuver. During the initial stages of the maneuver, due to the large deviation of the initial states from the switching surfaces and the large values of α_{1i} , the saturation constraints on the controls are active. Once the switching surfaces are reached and the sliding motion is initiated, the control magnitudes rapidly decrease. Figure 5 shows the magnitudes of the switching functions S_i . Figures 6 and 7 show the magnitudes of the switching functions and the control torques, respectively, during the terminal maneuver. The state trajectory remains well inside the deadband. The control torque

magnitudes are less than 0.05 N·m, suggesting that reaction wheels can be brought on-line.

Conclusion

The large-angle maneuver problem is treated using concepts of variable-structure control theory. A control law that constrains the state trajectory to follow a sliding mode is designed, based on a simplified spacecraft model. The sliding motion is shown to be optimal in the sense of a quadratic performance index in the Euler parameters and angular velocities. Simulation results indicate that the control system performs acceptably in the presence of unmodeled dynamics, disturbance torques, and parameter variations. It is assumed that the thruster control system includes both continuous and pulse-mode operating capabilities to provide the demanded torques.

Acknowledgment

This work was supported by the Engineering Research Institute of Iowa State University. The suggestions of the reviewers and the associate editor, Dr. F. Landis Markley, are appreciated.

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